Skin Friction Patterns for Transitional Flow in Annuli

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Main stream velocity profiles have been obtained by means of impact probes for the steady, isothermal flow of water in three smooth, concentric annuli having widely different diameter ratios. The point of maximum local velocity has been determined, thus permitting the ratio of skin frictions at the inner and outer boundaries to be calculated. Previously published data on pressure drop have been used to obtain separate friction factor correlations for the two surfaces. Attention has been centered on the transition range, where the position of maximum velocity is a function of both the diameter ratio and the Reynolds number.

Not much is known about the transitional behavior of fluids flowing in noncircular ducts, especially those in which the profiles of shearing stress and velocity are asymmetric about their minimum or maximum points. The concentric annulus is such a configuration and offers the advantage for study of uniform but different skin frictions on its two boundaries. The present work seeks to determine the separate transitional skin frictions at the inner and outer surfaces as functions of the Reynolds number and diameter ratio. For this purpose temporal mean, main stream, local velocities, as averaged by an ordinary impact probe, are experimentally measured in order to determine the radius of maximum velocity. With the radial position of zero shear thus established, previously published frictional data permit the individual skin frictions to be calculated.

If the inner radius of an annulus is r_1 and the outer radius is r_2, the corresponding skin frictions τ_1 and τ_2 are uniform over their respective boundaries but differ from one another in a definite fashion. Under steady flow conditions a simple force balance on the fluid shows that

$$\frac{\tau_1}{\tau_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{r_m^2 - r_1^2}{r_2^2 - r_m^2}\right) \qquad (1)$$

regardless of whether the flow is viscous or turbulent.

For fully viscous, isothermal flow it can be shown that

$$r_m^2 = (r_2^2 - r_1^2)/\ln(r_2^2/r_1^2) \qquad (2)$$

The ratio of skin frictions is therefore fixed by geometrical factors alone. Whether the same is true for transitional and fully turbulent flow however remains a matter to be determined through experiment. Several investigators (*1*, *4*, *9*) have concluded that the radius of maximum velocity is essentially independent of Reynolds number in fully turbulent flow and is given by Equation (2) within experimental error. In at least one instance however disagreement with this view has been reported (*2*).

In the region of viscous-turbulent transition skin friction measurements (*5*) and velocity data (*9*) have indicated that the radius of maximum velocity depends on the Reynolds number, but the influence of diameter ratio has not been established. The present experiments extend the previous work by dealing with a much wider range of diameter ratios.

With the radius of maximum velocity experimentally determined the skin friction ratio can be obtained from Equation (1). It is more usual however to express results in terms of the Fanning type of friction factors defined by means of the following equations:

$$f_1 = 2\tau_1 g_c/\rho V^2 \qquad (3)$$

$$f_2 = 2\tau_2 g_c/\rho V^2 \qquad (4)$$

Since the friction factors are defined on the basis of the average velocity over the whole stream, their ratio is equal to the ratio of skin frictions provided the density of the fluid is constant. Thus for the case at hand

$$f_1 = f_2\left(\frac{r_2}{r_1}\right)\left(\frac{r_m^2 - r_1^2}{r_2^2 - r_m^2}\right) \qquad (5)$$

The friction factor f_2 can be obtained from the pressure drop measurements of Walker, Whan, and Rothfus (*10*). Therefore f_1 can be computed and correlated as a function of Reynolds number and diameter ratio.

There are several Reynolds numbers that might be chosen as the basis of correlation. For instance one could be defined in such a way as to eliminate the effect of diameter ratio from the correlation of f_2 in the fully turbulent range of flow. However the Reynolds number

$$N_{Re} = (D_2 - D_1)V\rho/\mu \qquad (6)$$

is simple to compute, independent of the radius of maximum velocity, and the one most commonly used. Consequently it has been chosen as the basis of correlation in the present work.

EXPERIMENTAL EQUIPMENT

The apparatus was essentially the one used by Walker and Rothfus (*9*) with only minor modifications. The piping system permitted recirculation of water at about room temperature through an annular test leg 30 ft. in horizontal length. The annulus was supported by a heavy steel framework, insulated against vibration, and fitted with adjusting screws for vertical and horizontal alignment of the conduit. The cores extended through the ends of the test leg and were kept concentric by means of controlled tension as described by Walker (*8*). It was established by means of a cathetometer that eccentricity of the core within the test section did not exceed 0.001 in.

The outer tube was of smooth brass with an inner diameter of 0.750 in. Three cores were used, one of copper tubing with an outer diameter of 0.375 in. and two of stainless steel wire with outer diameters of 0.148 and 0.0465 in. The diameter ratios of the experimental annuli were therefore 0.500, 0.197, and 0.062. In addition Walker's data for a comparable annulus with a diameter ratio of 0.331 were included.

The outer brass tube was replaced by a 3-ft. section of Lucite tubing midway along the test leg. The Lucite tube had the same diameters as the brass tube and was aligned with it through machined joints. The static pressure tap and impact probe entered opposite sides of the Lucite tube. The probe was a Monel tube of 0.058 in. O.D. with its end closed by a thin cap and an impact opening 0.033 in. in diameter drilled adjacent to the cap. The probe was installed at a point along the test leg which was 274 to 485 equivalent diameters from the entrance depending on the size of the core. The corresponding downstream distance to the exit was between 209 and 369 equivalent diameters. On the basis of previous experiments (*5*) these distances are believed to have provided adequate calming length. The impact probe was attached to a micrometer type of traversing mechanism which was capable of positioning the center of the impact opening with a precision of better than 0.001 in.

Pressure differences were measured by means of manometers containing either

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monochlorobenzene or monofluorobenzene under water. At the highest multiplication pressure differences as small as 0.0001 in. of water could be detected with the aid of a cathetometer. A silicone fluid was baked on the walls of the manometer tubes to prevent the meniscus from sticking.

Flow rates were determined volumetrically by means of a calibrated tank. Temperature control was maintained within ±0.1°C. by circulating a portion of the water through a heater or cooler when necessary.

EXPERIMENTAL PROCEDURE

Prior to each day's run the concentricity of the core and the equilibrium positions of the manometers were checked. New manometer tubes were installed whenever periodic checking indicated that the meniscus was not behaving properly. Once the desired flow rate for any run was established, at least an hour was allowed to pass before any local velocities were measured.

The order of positions taken by the impact probe was changed from time to time, but no effect of sequential positioning was noted. The probe was ordinarily moved across the annulus in increments of 0.020 in. except near the point of maximum velocity where readings were taken every 0.001 or 0.002 in.

The impact probe was calibrated in the manner of Stanton, Marshall, and Bryant (7), with Lamb's theoretical profile (3) as the basis of comparison. With the calibration curves established, velocity measurements were made at various Reynolds numbers. For any single traverse the ratio of local to average velocity u/V was thus determined as a function of the radial distance r from the center of the annular configuration.

Only mainstream velocities were measured, since attention was focused on the point of maximum velocity and the probe was not designed for use in the region very close to the walls. In order to get a rough check on the experimental points however they were extrapolated to the walls with the aid of the theoretical velocity equations for viscous flow. The quantity $r(u/V)$ was then plotted against r between the radial limits r_1 and r_2. The area under the curve was compared with $0.5(r_1^2 + r_2^2)$, the correct value predictable from the definition of the bulk average linear velocity. For all Reynolds numbers the standard deviation of the integrated average velocity from the measured one was 2.7% and the average deviation was +0.33%.

After sufficient data were obtained, a graph of the radius of maximum velocity against Reynolds number was drawn at each diameter ratio. Additional values of the maximum velocity u_m were then measured without recourse to a complete velocity profile, since the impact probe could be situated immediately at the approximately correct point in the stream.

EXPERIMENTAL RESULTS

Figure 1 presents a dimensionless correlation of the radius of maximum velocity as a function of the Reynolds number at constant values of the diameter ratio. The data of Walker and Rothfus are also included. The horizontal, broken lines represent the theoretical value of the ordinate in fully viscous flow at each diameter ratio.

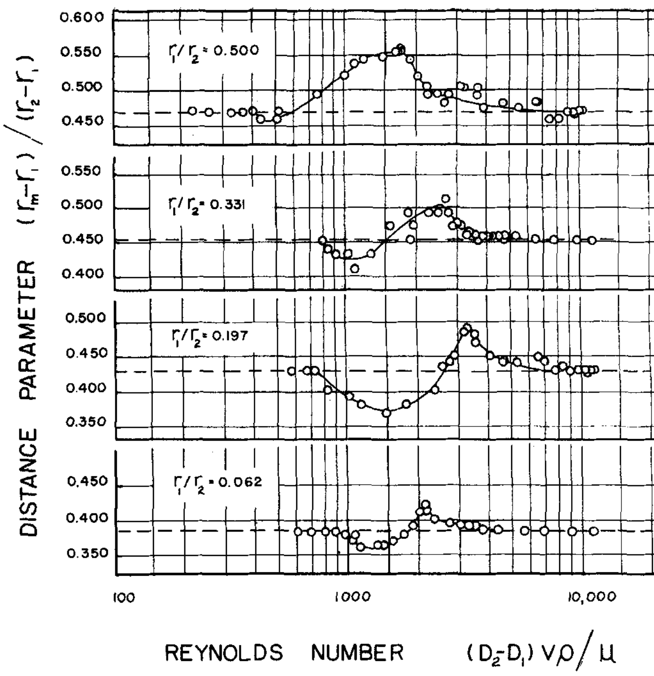

Fig. 1. Effect of Reynolds number on the radius of maximum velocity.

Figure 2 illustrates the relationship among the friction factors and Reynolds number at constant diameter ratio. Values of f_2 are taken from the data of Walker, Whan, and Rothfus (10). The corresponding values of f_1 are computed by means of Equation (5) with radii of maximum velocity from the solid lines shown in Figure 1.

Figure 3 shows the dependence of the ratio of average to maximum velocity on Reynolds number at constant diameter ratio. The theoretical value of the ordinate for fully viscous flow is shown by the broken horizontal line at each diameter ratio. The maximum velocity is the measured value at the actual maximum point, and the average velocity is the one measured by means of the calibrated tank.

DISCUSSION OF RESULTS

The data of Figure 1 indicate that the radius of maximum velocity varies with both diameter ratio and Reynolds number in the region between fully viscous and fully turbulent flow. It has been pointed out previously (9) that the data for the annulus having a diameter ratio of 0.331 are consistent with drag measurements on the core of a comparable annulus (5).

In the fully viscous region there is close agreement with the theoretical value of the radius of maximum velocity shown in Equation (2). In the fully turbulent region the experimental points suggest that the radius of maximum velocity attains its viscous-flow value within experimental error. Whether a true independence of Reynolds number can be claimed for the whole turbulent range is a matter which requires additional experimentation. The present data show no tendency for the radius of maximum velocity to vary with Reynolds number over the lower turbulent range covered in the experiments.

Through rearrangement of Equation (2), it can be shown that the ratio of skin frictions decreases as the radius of maximum velocity decreases in a specific annulus. Thus the initial movement of the maximum point toward the core tends to equalize the skin frictions, and the outward movement, at some higher Reynolds number, tends to increase their ratio. The mechanism through which the shift in maximum point occurs remains undetermined. A thorough study of the transients in transitional flow will probably be required in order to shed much light on the subject.

Koch and Feind (2) have reported that pressure drop and drag measurements in several annuli show that the skin friction ratio depends on the Reynolds number as well as on the geometry over the fully turbulent range. Their ratios fall from the fully viscous value to unity at the onset of transition and then gradually increase as the Reynolds number is raised. The viscous value is again attained at Reynolds numbers in the order of 100,000.

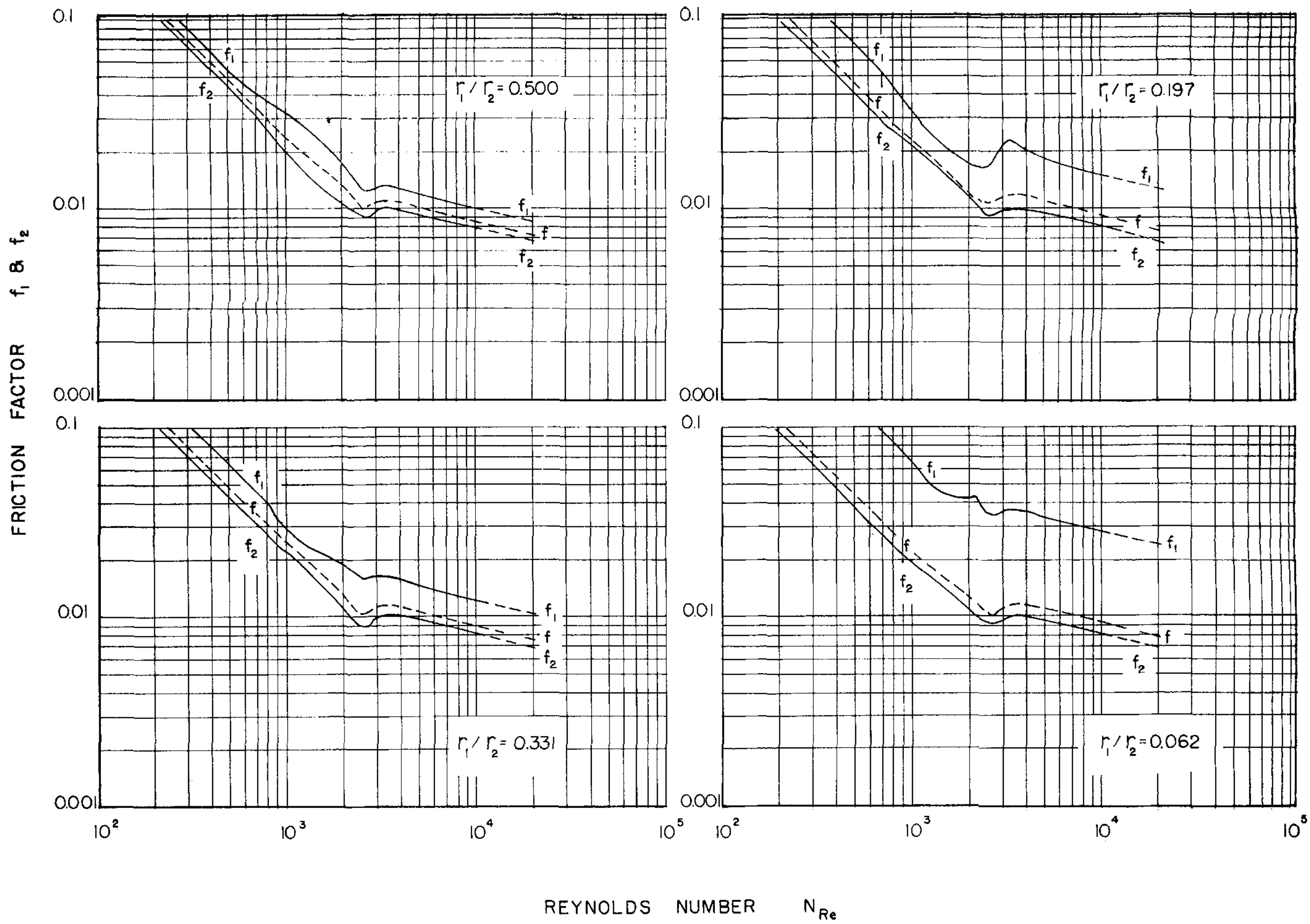

Fig. 2. Correlation of friction factors: f_1 = core [Equation (3)], f_2 = outer wall [Equation (4)], and f = over-all [Equation (8)].

This indicates an initial inward shift of the maximum point and a progressive outward movement at higher Reynolds numbers. Since the details of their experimental work are not presented, the results of Koch and Feind must, for the time being, be acknowledged to differ from those of the present work, but a full evaluation of the difference will have to be postponed until more data are available.

The impact tube was calibrated at several Reynolds numbers in the laminar range by obtaining velocities at known positions in the fluid stream on the assumption that the probe behaved ideally. The measured velocities were then brought to coincidence with the theoretical profile for the same flow rate through adjustment of the radial dimension. The calibration curves for each annulus thus compared the observed distance from the wall of the outer tube to the center of the impact opening with the effective distance.

As noted in previous work with tubes and annuli the influence of Reynolds number on the calibration curves was negligible, and the greatest correction was needed very close to the walls.. In all of the annuli used in the present work the effective and observed distances differed by 0.002 in. or less over a radial interval of about 0.070 in. roughly centered on the point of maximum velocity. The average spread of points on the calibration curves was ±0.002 in. over this part of the stream, which was the part most important to the present investigation.

Only mean velocity components in the axial direction were measured; secondary motions such as might arise from slight eccentricity of the core were not investigated. It should be emphasized that the data show only the average behavior of the velocity profile as determined by the total head type of probe. The question remains open as to how closely the true temporal mean local velocity is indicated by such an instrument, especially in the transitional regime where turbulent flashes occur intermittently over relatively large increments of time and space. It is known however that in the fully turbulent regime impact probes calibrated by the method of Stanton, Marshall, and Bryant (7) in the laminar range yield point velocities which are in satisfactory agreement with those obtained by taking measurements with probes of different diameters and extrapolating to zero probe size.

The friction factor correlations reflect the variation of the radius of maximum velocity with Reynolds number at a given diameter ratio. The friction factor for the outer wall f_2 is computed from pressure drop data through the modified Fanning equation

$$f_2 = \frac{\Delta p g_c (r_2^2 - r_m^2)}{r_2 \rho V^2 L} \qquad (7)$$

which is consistent with the definition of f_2 shown in Equation (4). Since f_1 in turn, is calculated by means of Equation (5), both f_1 and f_2 depend on the measured radius of maximum velocity. Consequently when these are plotted against a Reynolds number which does not depend on the maximum point, the individual friction factor curves are influenced by the combined effects of the pressure gradient and the position of maximum velocity. On the other hand the familiar over-all friction factor

$$f = \frac{\Delta p g_c (D_2 - D_1)}{2 \rho V^2 L} \qquad (8)$$

does not depend on the radius of maximum velocity and therefore reflects the behavior of the pressure gradient alone. The over-all friction factor is shown by the broken lines in Figure 2. Although the changing maximum point seems to influence the

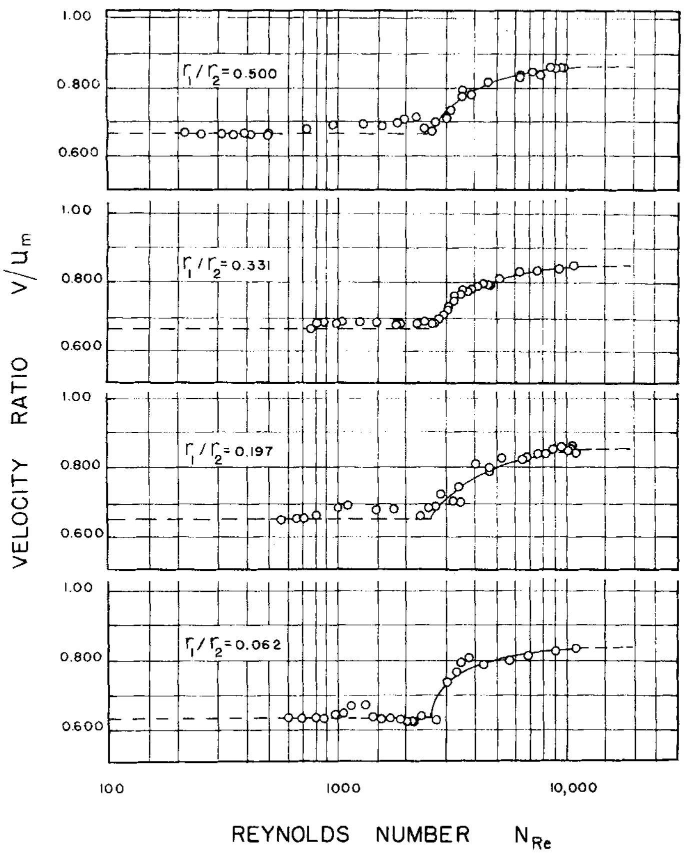

Fig. 3. Effect of Reynolds number on the ratio of average to maximum velocity.

pressure gradient slightly, the effect is almost within the limits of experimental error and therefore cannot be evaluated with certainty. There appears to be a tendency for the pressure gradient to decrease as the maximum point moves toward the core and to increase as it moves in the direction of the outer wall.

The pressure gradient and therefore the over-all friction factor f indicate the critical Reynolds number to be 2,500 to 2,600 for all of the investigated diameter ratios. On the same basis the critical Reynolds numbers for tubes and for parallel plates have been found to be 2,100 and 2,750 respectively. It should be noted that even an annulus with a small core is quite different from a tube, since the maximum point remains well removed from the center of the annular configuration unless the diameter ratio is extremely small. Thus it is not surprising to find the critical Reynolds number quite close to the one for parallel plates.

The friction factor at the outer wall appears to show the effects of the first distributed disturbance eddies at a Reynolds number of 2,600 to 2,700, about like the over-all friction factor. On the other hand the behavior of the core-side friction factor is more complicated. The changes in the skin friction ratio associated with the changes in the radius of maximum velocity are for the most part more apparent in f_1 than in f_2. There is always a transitional dip in the f_1 curve, as shown in Figure 2, but the dip is either reinforced or partially cancelled by the change in maximum point, depending on the diameter ratio. There is a tendency for the core-side fluid to reach the bottom of the dip and begin its transitional rise at a Reynolds number of 2,300 to 2,600, somewhat smaller than the corresponding one for the fluid outside the maximum point.

The behavior of the core-side friction factor in the annulus having a diameter ratio of 0.062 is noteworthy. In this case there is a peak at a Reynolds number of 2,100 which corresponds to the outward swing of the maximum point shown in Figure 1. The data do not justify other than a smooth curve of f and f_2 in this region, so the increase in skin friction ratio is entirely absorbed by f_1. In fact the core-side friction factor rises to about the value that might be obtained by extrapolating the curve for the fully turbulent range back to the Reynolds number of 2,100. This raises the question of whether a quasi turbulent situation first develops in the fluid on the core side of the maximum point. Since the mechanism cannot be established from the present data, the answer must await further information. Whether associated with intermittent turbulence or a more ordered motion, the skin friction ratio at the peak is not maintained at greater Reynolds numbers where the whole stream is influenced by turbulent eddies. Instead the ratio decreases toward its fully viscous value as indicated implicitly by the curve in Figure 1.

The ratio of average to maximum velocity is in close agreement with the theoretical value over the fully viscous range of flow. Once the shift in maximum point occurs however, the ratio increases by a few per cent. Although the changes are too small to be established with certainty, there is a tendency for the ratio to increase with the inward movement of the maximum point and to decrease somewhat with its outward movement. When transition is reached, the ratio increases with Reynolds number in the usual way. At a Reynolds number of 10,000 the measured ratios are from 0 to 2% above those predicted by Rothfus, Walker, and Whan (*6*) from their generalized correlation of local velocities in fully turbulent flow. This is within the combined errors of the present results and of the data on which the generalized correlation is based.

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NOTATION

D_1 = diameter of core, ft.
D_2 = diameter of outer tube, ft.
f = over-all friction factor defined in Equation (8), dimensionless
f_1 = friction factor for core defined in Equation (3), dimensionless
f_2 = friction factor for outer wall defined in Equation (4) dimensionless
g_c = conversion factor = 32.2 (lb.-mass) (ft.)/(lb.-force) (sec.^{2})
L = length of conduit, ft.
N_{Re} = Reynolds number defined in Equation (6), dimensionless
p = pressure drop due to friction, lb.-force/sq. ft.
r = radial distance from center of annular configuration to point of measurement, ft.
r_1 = radius of core, ft.
r_2 = radius of outer tube, ft.

r_m = radius to point of maximum local velocity, ft.
u = temporal mean local velocity of fluid at radius r, ft./sec.
u_m = maximum temporal mean local velocity of fluid at radius r_m, ft./sec.
V = bulk average linear velocity of fluid, ft./sec.
μ = viscosity of fluid, lb.-mass/(sec.) (ft.)
ρ = density of fluid, lb.-mass/cu. ft.
τ_1 = skin friction at core, lb.-force/sq. ft.
τ_2 = skin friction at outer wall, lb.-force/sq. ft.

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Heat Transfer from the Surface of a Steam Bubble in Turbulent Subcooled Liquid Stream

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Turbulent heat transfer coefficients have been measured at the surface of single bubbles formed by injecting steam into a subcooled water stream at atmospheric pressure. Depending upon the steam flow rate (0.4 to 1.5 g./min), the water temperature (80° to 180°F.), and the water velocity (0.9 to 7.2 ft./sec.) the bubbles ranged from small, smooth, ellipsoidal bubbles, similar to those observed in highly subcooled nucleate boiling, to large, irregular bubbles which oscillated in size. The bubble frequencies were in the range 200 to 2,500 cycles/sec. and the surface heat transfer coefficients 13,000 to 320,000 B.t.u./(hr.) (sq. ft.) (°F.). Because of these exceptionally high heat transfer coefficients a significant fraction of the total heat flow in Gunther's subcooled boiling experiments is estimated to be attributable to latent heat transport.

A problem of considerable importance in the study of the mechanism of nucleate boiling transfer is whether an appreciable fraction of the total heat flux flows in the form of latent heat through the two-phase wall boundary layer into the turbulent core. Gunther and Kreith (*1*, *2*) and also Rohsenow and Clark (*3*) measured the rate of production of vapor in subcooled nucleate boiling of water at atmospheric pressure and noted that only 1 to 2% of the total heat flux in subcooled nucleate boiling could be accounted for by the latent heat content of this vapor. Gunther and Kreith further considered the possibility of simultaneous vaporization of hot liquid at the base and condensation at the top of the bubble and showed that this would not greatly increase the rate of latent heat transport, providing that the bubble is surrounded by a stagnant liquid film. The tentative conclusions which were drawn were that the rate of latent heat transport was negligible, the bubbles acting principally as small stirring devices. Essentially this would imply that it would make no difference whether the bubbles were filled with vapor or with inert gas, and indeed it has been shown by Beatty and co-workers (*4*) that nonboiling heat transfer coefficients are increased by a factor of 2 or 3 by generating inert gas bubbles at the heating surface. However the heat fluxes produced, even at the highest generation rate of inert gas bubbles, were in the range of the lower limit of nucleate boiling. One may therefore speculate that, although the stirring effect dominates relatively low heat fluxes, another mechanism may become important as the bubbles become more closely packed and the upper limit of nucleate boiling is approached. Bankoff (*5*), in a study of the mechanism of subcooled boiling, noted several pieces of indirect evidence which indicated that latent heat transport was not insignificant, especially near the upper limit of nucleate boiling, and concluded that the relative importance of stirring and latent heat transport effects needed further study. It was suggested that turbulent heat transfer coefficients at the surface of a rapidly growing bubble might be one or more orders of magnitude greater than the heat transfer coefficient to a bubble surrounded by a liquid in laminar radial flow with no tangential components. The present work, which is a study of turbulent heat transfer coefficients to rapidly growing and collapsing bubbles formed at the surface of a horizontal plate in the presence of a subcooled liquid stream, was undertaken to provide further information on this subject.

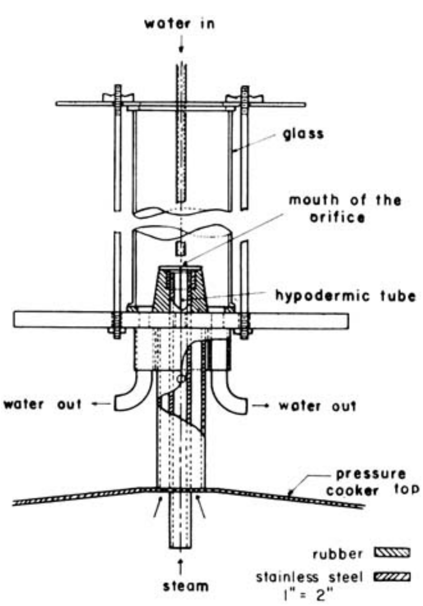

Fig. 1. Bubble heat transfer cell.

PREVIOUS WORK

Although a number of studies of heat and mass transfer to solid spheres and droplets have been made, no previous study has come to the attention of the authors of heat transfer to